(Lecture 06) Loan Survival Rate

JOSHUA KAVNER

1 Survival analysis

In the last lecture, we discussed a homeowner, Jon, who took out a mortgage and paid back the loan with a fixed amount each month for 30 years. This was captured by the following recurrence relation:

$$a(t+1) = a(t)\left(1 + \frac{r}{12}\right) - P \tag{1}$$

where $a(t)$ is loan balance, r is the annualized interest rate, and P is the monthly payment. During 360 time periods, Jon's net balance reduced to zero: $a(0)$ is the initial loan amount, and $a(360) = 0$. We therefore found that Jon's monthly payment was

$$P = \frac{a(0)u^{360}(1-u)}{1-u^{360}}$$

where $u = 1 + \frac{r}{12}$. Looking at Equation (1), it is clear that Jon must pay

$$P \geq a(t)\,(1 + rT) \tag{2}$$

in order to shrink his balance over time, where T is denoted in fractions of years. The right-hand side of Equation (2) is the interest accrued over the period T by the outstanding balance $a(t)$, so as long as Jon pays more than this, the net balance will decrease. Otherwise, the net balance will increase at an exponential rate.